**DIFFERENTIAL EQUATIONS**

**DIFFERENTIAL EQUATIONS:**

The equation contains independent variable (), dependent variable () and it’s derivative ().

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| **ORDINARY DIFFERENTIAL EQUATION (ODE)** | **PARTIAL DIFFERENTIAL EQUATION (PDE)** |
| ODEs are formed, When | PDEs are formed, When |

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| **ORDER OF EQs.** | **DEGREE OF EQs.** |
| Highest order derivative term present in the equation. E.g. 1st order, 2nd order, 3rd order. | Power of Highest order derivative term in the equation. E.g. is 2nd Degree Equation. |

If there is fraction in power, eliminate fraction and make whole number.

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| **ORDINARY DIFFERENTIAL EQUATIONS (ODEs)** | |
| **FIRST ORDER DIFFERENTIAL EQUATIONS** | **HIGHER ORDER DIFFERENTIAL EQUATIONS** |
| 1st order & 1st degree (Linear) D.E.,   1. Variable Separable Form, 2. Homogenous D.E. 3. Linear D.E. 4. Exact D.E. | Higher order & 1st degree (Linear) D.E.,   1. With Constant coefficients 2. With Variable coefficients |

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| **SOLUTION OF A DIFFERENTIAL EQ.:**   * Relation Between dependent & independent variables * Should not contain derivative/ differential term * Solution must satisfy given D.E. | **TYPES OF SOLUTIONS** | |
| **General Solution** | **Particular Solution** |
| Contains Arbitrary Constant | Don’t Contains Arbitrary Constant. E.g. BCs are given |

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| **VARIABLE SEPARABLE FORM** | **EQUATIONS REDUCIBLE TO VARIABLE SEPARABLE FORMS** | |
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| * Separate the variable, * Integrate Both Side, * Add appropriate Constant, |  | Limiting Condition: |

**HOMOGENOUS FUNCTION:**

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| **HOMOGENOUS DIFFERENTIAL EQUATIONS** | |
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| Where, both are homogenous equations with same degree. | |

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| **LINEAR/ LABNITZ’S** **DIFFERENTIAL EQUATIONS** | | | |
| Dependent Variable and it’s derivative are not multiplied together. | | | |
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| Where, are function of independent variable. | | | |
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| **EXACT DIFFERENTIAL EQUATIONS** | |
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| **REDUCING EQUATION TO EXACT FORM** | | |
| **FORM-I:** If is not exact differential equation but it’s homogeneous differential equation, multiply equation with Integrating factor to get Form Exact DE, | **FORM-II:** If is not exact differential equation, multiply equation with Integrating factor to get Form Exact DE, | |
| **FORM-III:** If is not exact differential equation, | | |
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| **INTEGRATING FACTORS BY INSPECTION** | | | | | | | |
| **Sr. No.** | **Group of Terms** | **I.F.** | **Exact Differential** | **Sr. No.** | **Group of Terms** | **I.F.** | **Exact Differential** |
| 1 |  |  |  | 6 |  | 1 |  |
| 2 |  |  | 7 |  |  |
| 3 |  |  | 8 |  |  |
| 4 |  |  | 9 |  |  |
| 5 |  |  | 10 |  |  |

**HIGHER ORDER LINEAR DIFFERENTIAL EQUATION:**

**WITH CONSTANT COEFFICIENTS:**

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|  | | | Where, are constant coefficients,  is dependent and is independent variable. |
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|  | Complete Solution always denote by dependent variable | | |

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| **METHOD TO FIND CF** | |
| Step 1: Write the given In D-Form. | Step 2: Write Auxiliary (AE) Equation, |
| Step 3: Solve Auxiliary (AE) Equation & find it’s roots. | Step 4: Write CF according to nature of roots. |

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| **CASE-I:** Roots are Real and Distinct | **CASE-II:** Roots are Real & Equal |
|  |  |
| **CASE-III:** Roots are Complex | **CASE-IV:** Roots are Complex & Equal |
| For , | For , |

If in , .

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| **METHOD TO FIND PI** | |
| 6 BASIC CASES | METHOD OF VARIATION OF PARAMETERS |

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| **CASE** | **VALUE OF X** | **REMARKS** |
| I |  |  |
| II |  |  |
| III |  | is positive integer |
| IV |  |  |
| V |  |  |
| VI |  | & is positive integer >1 |

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**CASE I:** Replace by .

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**CASE II:** Replace by .

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**CASE III:** Use binomial Expansion

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| **CASE IV:** | **CASE V:** |
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**CASE VI:**

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**METHOD OF VARIATION OF PARAMETERS:** Use when none of above useful & Used for 2nd order LDEs.

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| Step 1: Find CF  Step 2: Compare with  Step 3: Find . Also Find .  Step 4: Find Common Term | Step 5: Find  Step 6: Find  Step 7: |

**WITH VARIABLE (INDEPENDENT VAR) COEFFICIENTS:**

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| Cauchy’s Homogenous form | Legendre’s Homogenous form |

**CAUCHY’S HOMOGENOUS FORM:**

|  |  |  |  |  |
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**LEGENDRE’S HOMOGENOUS FORM:**

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Here, z is independent variable.

**FIRST ORDER PARTIAL DIFFERENTIAL EQUATIONS**

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| 1st Order | 1st Order | 2nd Order | 2nd Order | 2nd Order |

**1ST ORDER PDEs:** The equation contains only terms.

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| **1ST ORDER PDEs** | |
| **Linear 1ST Order PDEs** | **Non-Linear 1ST Order PDEs** |
| If are with degree one and are not multiplied together are linear 1st Order PDEs. | If are not with degree one or are multiplied together are Non-Linear 1st Order PDEs. |

**LINEAR 1ST ORDER PDEs:**

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| **LAGRANGE'S EQUATION** |  |  |

**NON-LINEAR 1ST ORDER PDEs:**

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| **TYPE-I:** | **Sol:** | Where, relate with relation |

**CONCEPT:** Non-Linear 1st Order PDEs can only contain 2 constants.

|  |  |  |  |  |  |  |  |  |  |
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| **TYPE-II:** , | | **be a trial solution also,**   |  |  |  | | --- | --- | --- | |  |  |  | | Initial Variable | Intermediate Var. | Final Variable | | |
|  |  | |  |
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| **TYPE-III:** When Eq. has such that, | Find from given conditions. |

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| **TYPE-IV:** Clairaut’s form, |  |

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| **2ND ORDER PDE (TYPE IDENTIFICATION)** | | | |
|  | **TYPE OF PDE** | | |
| **ELLIPTICAL** | **PARABOLIC** | **HYPERBOLIC** |
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| **WAVE EQUATION** | **HEAT EQUATION** | **LAPLACE EQUATION** |
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| **METHOD OF SEPARATION OF VARIABLE** |